The risk associated with a mining project comes from the uncertainties involved in the industry. Mining companies endeavouring to maximize their return for shareholders make important strategic decisions which take years or even decades to “play out”. However, many mining companies feel comfortable with point estimates of all project parameters but realize that no parameter value is known with certainty. A model that incorporates uncertainties and is able to adapt will help deliver a design with a better risk-return profile. In this paper, a new methodology is developed in order to have a design that is flexible and able to adapt with change. Following recent research on decision making methods in mine planning, this paper develops a mixed integer programming model that determines the optimal design for simulated stochastic parameters. The paper shows how to incorporate optionality (flexibility) in relation to mine, stockpile, plant and capacity constraint options. Obtained results are promising and are helping decision makers to think in terms of value, risk and frequency of execution.

**Real options, robust design, uncertainty, flexibility, stochastic simulation, mine design**

**INTRODUCTION**

Mining projects are characterised as being highly uncertain and risky due mainly to the volatile nature of commodity prices, as well as the inherent uncertainty around geological conditions. As risk in the mining project cannot be eliminated, the best that can be done is to minimize it. The uncertainties in mining projects arise from both nature of the variables and cost of obtaining information about them. They can come from many different sources including market prices, grade distribution, ground conditions, equipment reliability, recovery of ore, human capital and legislative change [1 – 4]. The mining industry will be more sustainable if projects were developed in a manner that increases flexibility to respond to uncertainties. For example, the minerals industry in the last decade saw unprecedented demand for its products which lead to a flurry of expansion activity, however no one predicted the global financial crisis which lead to a rapid decline in expansion activity. Had this information been known prior to these events occurring it is likely the path taken would have been vastly different. Being able to design an operation that has flexibility to respond to change quickly should deliver better returns to stakeholders.

Geological uncertainty, e.g. [5, 6], and risk have been incorporated in optimum mine planning and design by a few studies to date. Godoy and Dimitrakopoulos [7] developed a multistage optimization approach, based on a simulated annealing algorithm, for long term mine scheduling under geological uncertainty. In the case study a 28% higher NPV was generated compared to conventional methods, whilst also reducing the risk in meeting production targets. Similarly, a 26% NPV increase was obtained in recent studies by Leite and Dimitrakopoulos [8] by utilizing a variant of the same method. Ramazan and Dimitrakopoulos [9] developed a stochastic integer programming (SIP) model to generate the optimal production schedule using equally probable stochastically simulated orebody models as inputs. The proposed approach minimizes the risk of not meeting production targets as a
function of ore, metal and grade blending. Two applications of this methodology have showing an expected total NPV increase of ten and twenty five percent using a SIP model over a traditional schedule based on single estimated orebody. Leite and Dimitrakopoulos [8] use the same SIP formulation to generate risk-robust solution. The same copper deposit has been utilized in [10] and the stochastic formulation is shown to produce 29% higher NPV than the schedule obtained from a conventional scheduler. Meagher et al. [11] introduced an approach to integrate “block destination flexibility” in the process of assigning value to mining blocks in the planning process via real options valuation (ROV) considering geological and market uncertainties. The proposed approach; firstly assigns a dollar value to each mining block, considering the different aspects of uncertainty and management flexibility. Secondly, it utilizes a minimum cut algorithm to design a lower risk long-term mine plan. Applications of this method to a case study demonstrated significant differences in block value estimates then conventional approaches.

A decision making tool, called “real options “in” projects” (ROIP), has been developed to increase the flexibility of an engineering system under uncertainties. This method is located midway between financial real options analysis (which does not deal with engineering system flexibility) and traditional engineering approaches (which do not deal with financial flexibility). ROIP benefits by being able to adjust the underlying system in response to the resolution of uncertainties over time. Significant research into this method has been undertaken by de Neufville and his colleagues [12] with applications in various industries. A frequently used example to explore the concept of ROIP is that of a multi-story car park. Flexibility in this situation is in the design of the footing and columns of the building so that additional levels can be added at a later date. This flexibility comes at a cost, and the designer must determine if this is warranted [13]. Another example provided by Wang and de Neufville [14, 15] is that of the bridge over the Tagus River in Lisbon, Portugal. The original design of the bridge was ‘enhanced’ by including an allowance to build a second deck above the first. This was achieved by increasing the size of the initial footings. These additions to the design resulted in considerable additional cost. Subsequently the Portuguese government exercised their option to expand the bridge and added a second deck to carry a suburban railroad.

In [13] R. de Neufville implemented this technique into mining projects with a Chilean mine in the “Cluster Toki region. In the paper, a methodology using ROIP is implemented where different operating plans vary by truck fleet capacity and crusher size in response to changing prices. The application of this method resulted in approximately 30 to 50% more project value than current estimates. Even though this approach provides a strong basis on which to grow ROIP theory for mining, there are several deficiencies in the current model. The approach limits the flexibility up front by initial static scenario construction, fails to deal with variation in ore grade and recovery, and fails to consider options in all stages of a typical mine value chain. Further information about ROIP applications can be obtained from following references [14 – 17].

Groeneveld et al [18] outlined a methodology for a new flexible mine design methodology. Under this methodology options could dynamically be included in the design to represent mining, plant and port constraints. This model was then processed multiple times with each run representing a different ‘state of the world’. The limitations of this model however was; the inability to model multiple capacity constraints in a flow path, lack of flexibility to include multiple circuits in the processing plants, only one product could be produced and only simple network designs could be created (i.e. mine to stockpile or/plant to port). The paper seeks to fix these short comings by providing a mathematical model that can incorporate more flexibility, hence incorporate more design options.
This paper outlines a methodology to evaluate the flexibility of strategic mine design under uncertainty, using mixed integer programming (MIP) and Monte Carlo simulation (MCS). The methodology involves the incorporation of numerous design options (mine, stockpile, plant and port) and multiple uncertainties (price, capital cost, operating cost, recoveries and utilization). These factors are incorporated in a MIP model which solves for optimality. An application of this methodology to a copper-gold deposit will be undertaken in order to show the power of the model to handle complex strategic decisions.

**METHOD**

To evaluate the available flexibility in strategic mine design in order to determine beneficial options to execute, this research employs MCS and MIP. Uncertainties (or stochastic parameters) are simulated using MCS to generate inputs to a MIP model. The MIP model allows for “go” or “no go” decisions to be modeled for the optimal execution timing under a set of uncertainties. Running the model multiple times generates a database of optimal designs for given ‘states-of-the-world’. This dataset then provides a pathway to determine the flexibilities that provide the best risk-return profile.

**Design Options**

Four categories of options are available in the initial design phase that are incorporated dynamically into the model. These four categories are; mine options, pre-processing stockpile options, processing plants options and capacity constraint options. The objective is to determine the types of options used and the timing of their execution. Examining this will allow the decision maker to determine the best set of options to incorporate - when and at what capacity - in a mine plan.

The four types of options have different characteristics which need to be explained further. Mine options represent the physical extraction capacity that is required to move material from the ground. This constraint may be an annual tonnage constraint or an effective flat haul constraint which considers the time required to move material, useful in highly variable haul distance scenarios. Pre-processing stockpiles are stores of material after extraction from the ground, either for long term low grade scenarios, fluctuating demand scenarios or for waste material storage. Processing plant options represent the physical and/or chemical process that is undertaken to ‘recover’ ore from the gangue material. Processing plants may contain multiple different circuits which the material may pass through. These circuits may have different beneficiation characteristics. Capacity constraint options represent physical constraints which may need to be modeled in the design. These may represent things such as port capacity, loading facilities, crusher capacities or conveyor capacities. They may be incorporated at any point in the network.

**Resource Representation**

The representation of the resource in the model is carried out by parcels of material. A parcel of material can be defined as a quantity of material with an average grade determined by the weighted average of grade bins contained within the parcel. A parcel may be made up of one or more grade bins. A grade bin represents a quantity of material at a specified grade. These grade bins provide a higher resolution of data to the model, whilst minimizing the number of integer variables needed to provide this information. These parcels are designed to represent a physical constraint on the resource, such that they must be fully mined before mining a parcel lower in the physical sequence.
Flow Paths

A flexible mine design is created based on a set options that are incorporated in the model through a network structure. Different routes through the network are termed ‘flow paths’. A flow path is a singular route through the network that material could travel along; to explain this concept further consider Fig. 1.

Examples of flow paths in Fig. 1 include; the path from the resource (R) to mine 1 (M1) to stockpile 1 (S1) to plant 1 (P1) through circuit 1 (C1) to product A which would be RM1S1P1C1A, the path from the resource (R) to mine 1 (M1) to waste stockpile 1 (W1) which would be RM1W1, the path from the resource (R) to mine 1 (M1) to stockpile 1 (S1) to plant 4 (P4) through circuit 2 (C2) to product B (B) which would be RM1S1P4C2B and the path from the resource to mine 3 (M3) to stockpile 1 (S1) to plant 3 (P3) through circuit 1 (C1) to product A (A) which would be RM3S1P3C1A. This is only a small number of the potential paths through the network, in reality there are numerous flow paths.

An important aspect of the model formulation is that unique mine designs can be generated. That is capacity constraints can be incorporated anywhere along the network, multiple processing plants/routes can be included and products can be generated at any point in the network (helping to model options of selling the ore at the mine gate).

Stockpiling

Stockpiling is used in mine operations for many reasons including; blending of material, storage of excess mine production, storage of waste material and storage of low grade ore for future production. When material is stockpiled the grade and tonnage of the material is known. However, as the material is mixed on the stockpile the grade and the tonnage become unknown. Since the quantity of productive material in the stockpile is unknown prior to the optimization, this gives rise to a non-linear constraint. To solve this virtual grade bins are created in the stockpile. These grade bins have a maximum and minimum grade of material which can enter the bin. When removing material the average grade is taken from the grade or alternatively, the maximum or minimum grade limit of the bin can be used.

Stochastic Parameters

The model incorporates uncertainty around the input parameters by Monte Carlo simulation. Each simulation of these values represents a “state of the world” that is equally probable in the future. Various parameters can be incorporated in the model including; price, capital cost, operating cost, equipment utilization, recovery and time to build. Running a set of simulations is intended to give representative sample of the future “state of the world”.

Fig. 1. Example design option network showing numerous flow path options
MODEL FORMULATION

The developed MIP model optimizes the available design options for a simulated 'state-of-the-world'. Each design option can impact capital commitment, revenue generated and operating expenses. The optimization process seeks to determine the design and schedule with the highest net present value for the given financial and technical conditions. An outline of the mathematical formulation is provided below.

**Indices:**
- $r$ — rate of return on the project;
- $t$ — time period;
- $d$ — product type;
- $k$ — component of a product;
- $e$ — dependent options;
- $m$ — mining options within the set of design options;
- $s$ — stockpile options within the set of design options;
- $p$ — parcel of material;
- $b$ — a grade bin of material within a parcel;
- $n$ — bin of material within a stockpile;
- $f$ — flow path of material through the design network;
- $y$ — tolerance factor for the deviation of the mining of a bin within parcel;
- $c$ — circuit within a processing plant;
- $g$ — grade element of material within a resource.

**Parameters:**
- $P_{d,k,t}$ — the sale price of product $d$ component $k$ in time $t$ (in $$/metal unit);
- $V_{l,t}$ — the available resource of parcel $p$; $R_{p,b}$ — the available resource of parcel $p$ bin $b$; $R_{p+1}$ — the available resource of the successor parcel $p + 1$;
- $R_{k,e}$ — the recovery of material component $k$ through circuit $e$;
- $G_{g,k,f,t}$ — the metal units for grade $g$ in component $k$ in flow path $f$ in time $t$;
- $GL_{g,d,k}$ — the upper grade limit of grade $g$ product $d$ component $k$;
- $GU_{g,d,k}$ — the upper grade limit of grade $g$ product $d$ component $k$;
- $G_{g,k,s,t}$ — the grade $g$ to component $k$ through circuit $c$ in time $t$;
- $K_{l,t}$ — the capacity of option $l$ in time $t$;
- $K_{l,x,t}$ — the capacity of option $l$ circuit $c$ in time $t$;
- $E_{p,b,l}$ — the EFH required to move parcel $p$ bin $b$ to location $l$;
- $H_{l,t}$ — the EFH limit on option $l$;
- $DT$ — the lag time between these relationships (i.e. build option two, three periods after option one);
- $K_{n,t}$ — the stockpile capacity of stockpile $n$ in time $t$;
- $R_{k,s,n}$ — the calculated average, maximum or minimum recovery for all material of component $k$ in stockpile $s$ bin $n$;
- $G_{g,k,s,n}$ — the calculated average, maximum or minimum metal units of grade $g$ for component $k$ in stockpile $s$ in bin $n$;
- $D_{d,k}$ — the capacity of product $d$ component $k$ in time $t$;
- $GU_{g,d,k}$ — the upper grade limit of grade $g$ product $d$ component $k$;
- $GL_{g,d,k}$ — the lower grade limit of grade $g$ product $d$ component $k$;
- $GL_{n}$ — the lower grade limit of bin $n$;
- $GU_{n}$ — the upper grade limit of bin $n$.

**Variables:**
- $X_{l,t}$ — the tonnages through option $l$ in time $t$;
- $X_{p,b,l}$ — the tonnage mined from parcel $p$ in time $t$;
- $X_{p,b,f,t}$ — the tonnage from parcel $p$, bin $b$ through flow path $f$ in time $t$;
- $XR_{k,f,t}$ — the recovered tonnage to component $k$ through flow path $f$ in time $t$;
- $XI_{s,n,t}$ — the flow in from stockpile $s$ bin $n$ in time $t$;
- $X_{p,b,l}$ — the tonnage mined from parcel $p$ in time $t$;
- $XO_{s,n,t}$ — the flow out from stockpile $s$ bin $n$ in time $t$;
- $XR_{k,f,t}$ — the recovered tonnage to component $k$ through flow path $f$ in time $t$;
- $S_{d,k,t}$ — the sale quantity (tonnages or metal units) of product $d$ component $k$ in time $t$. 

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\begin{align*}
Y_{l,t} &= \begin{cases} 
1, & \text{if \ option \ } l \text{ \ is \ executed at \ time} \ t, \\
0, & \text{otherwise.} 
\end{cases} \\
Y_{p,t} &= \begin{cases} 
1, & \text{if \ parcel \ } p \text{ \ is \ fully \ mined \ at \ time} \ t, \\
0, & \text{otherwise.} 
\end{cases} \\
ID_{l,t} &= \begin{cases} 
0, & \text{if \ option \ } l \text{ \ is \ not \ disposed \ at \ time} \ t, \\
\text{otherwise} \text{ \ number \ of \ times \ disposed.} 
\end{cases}
\end{align*}

**Objective Function**

The objective function seeks to maximize before tax net present value (NPV):

\[
\sum_{t=1}^{T} \frac{1}{(1+r)^t} \left[ \sum_{d=1}^{D} \sum_{k=1}^{K} P_{d,k,t} S_{d,k,t} - \sum_{l=1}^{L} V_{l,t} X_{l,t} - \sum_{p=1}^{P} \sum_{b=1}^{B} \sum_{l=1}^{L} M_{p,b,l,t} X_{p,b,f,t} - \sum_{l=1}^{L} F_{l,t} Y_{l,t} - \sum_{l=1}^{L} D_{l,t}[f \in I] D_{l,t}[f \in I] + \sum_{l=1}^{L} F_{l,t}[f \in I] D_{l,t}[f \in I] \right].
\]

The constraints in the model can be divided into five categories: production, mining, stockpiling, processing and port constraints.

**Production Constraints**

*Resource Constraint.* This constraint makes sure the total amount of material extracted from a mining pit has an upper bound based on the resource. This constraint is applied at a parcel and bin level in the model:

\[
\sum_{i} X_{p,b,t} - R_{p,b} \leq 0 \quad \forall p, b.
\]

*Sequencing Constraint 1.* This constraint in conjunction with the next constraint forces the binary value to be one in the period the parcel is fully mined. This then allows the model to mine any successor parcel:

\[
\sum_{b=1}^{B} X_{p,b,t} \geq R_{p} \cdot Y_{p,t} \quad \forall p, t.
\]

*Sequencing Constraint 2.* This constraint ensures that a parcels predecessor is mined before the successor is mined:

\[
X_{p+1,b,t} \leq R_{p+1} \sum_{t=1}^{T} Y_{p,u} \quad \forall p, t.
\]

*Set Packing Constraint.* This constraint forces a parcel to only be mined once:

\[
\sum_{t=1}^{T} Y_{p,t} \leq 1 \quad \forall p.
\]

*Equal Mining Constraint.* This constraint ensures the equal mining of a parcel in each period within a given tolerance value ($\gamma\%$):

\[
\frac{1}{R_{p}} X_{p,t} - \frac{1}{R_{p,b}} X_{p,b,t} \leq \gamma\% \quad \forall p, b, t,
\]

\[
\frac{1}{R_{p}} X_{p,t} - \frac{1}{R_{p,b}} X_{p,b,t} \geq -\gamma\% \quad \forall p, b, t.
\]
Recovered Tonnage Constraint. This constraint calculates the recovered tonnage through a flow path.

\[ XR_{k,f,t} = \sum_{p=1}^{P} \sum_{b=1}^{B} R_{k,c} X_{p,b,f,t} \quad \forall k,f,t | c \in f. \]

Grade Units Constraint. This constraint calculates the grade units produced through a flow path.

\[ G_{g,k,f,t} = \sum_{p=1}^{P} \sum_{b=1}^{B} G_{g,k,c} X_{p,b,f,t}, \quad \forall q,k,f,t | c \in f. \]

Option Constraints

Option Capacity Constraint. This constraint applies the upper capacity limit for each option:

\[ X_{l,t} = \sum_{i=1}^{I} K_{l,i} Y_{i,t} + \sum_{i=1}^{I} K_{l,t} ID_{i,t} \leq 0 \quad \forall l,t. \]

Plant Circuit Capacity Constraint. This constraint applies a limit on the circuit capacity for the plant option:

\[ X_{l,c,t} = \sum_{i=1}^{I} K_{l,c,i} Y_{i,t} + \sum_{i=1}^{I} K_{l,c,t} ID_{i,t} \leq 0 \quad \forall l,t. \]

Mining Capacity Constraint. This constraint calculates the amount of capacity used for each mining option on an Effective Flat Haul Kilometres (EFH) basis:

\[ \sum_{p=1}^{P} \sum_{b=1}^{B} E_{p,b} X_{p,b,f,t} = \sum_{i=1}^{I} H_{i,t} Y_{i,t} + \sum_{i=1}^{I} H_{i,t} ID_{i,t} \leq 0 \quad \forall l,t. \]

Disposal Constraint. This constraint ensures that an option can only be disposed if it has previously been built:

\[ ID_{l,t} - \sum_{i=1}^{I} Y_{i,t} \leq 0 \quad \forall l,t \neq 1, \]

\[ \sum_{i=2}^{I} ID_{i,t} - \sum_{i=1}^{I-1} Y_{i,t} \leq 0 \quad \forall l,t \neq 1. \]

Option Dependency Constraint. This constraint ensures the dependent relationship between options exists. Execution of option \( Y_{l+1,t} \) depends on option \( Y_{l,t} \):

\[ Y_{l,t} - \sum_{n=1}^{N} Y_{l+1,t} \leq 0 \quad \forall l,t. \]

Stockpiling Constraints

Stockpile Capacity Constraint. This constraint ensures that the maximum stockpiling capacity is not exceeded in any time period:

\[ K_{n,t} \geq \sum_{i=1}^{I} X_{I_{s,n,t}} - \sum_{i=2}^{I} X_{O_{s,n,t}} \quad \forall s,n,t. \]

Stockpile Flow Out Constraint. This constraint ensures that the amount of material flowing out of a stockpile is less than or equal to what has flown in and flown out in previous periods:

\[ X_{O_{s,n,t}} \leq \sum_{i=1}^{I} X_{I_{s,n,t}} - \sum_{i=2}^{I} X_{O_{s,n,t}} \quad \forall s,n,t. \]
Stockpile Recovery Constraint. This constraint determines the recovered material from a stockpile processed through flow path $f$.

$$XR_{k,f,t} = \sum_{n=1}^{N} R_{k,s,n} X_{f,t} \quad \forall k,f,t \neq 1.$$  

Stockpile Grade Constraint. This constraint determines the grade produced from stockpiled material through flow path $f$:

$$G_{g,k,f,t} = \sum_{n=1}^{N} G_{g,k,s,n} R_{k,s,n} X_{f,t} \quad \forall g,k,f,t \neq 1 | s \in f.$$  

Product Constraints

Maximum Product Component Capacity. This constraint sets an upper bound on the amount of product that can be produced to a particular component:

$$S_{d,k,t} \leq D_{d,k,t} \quad \forall d,k,t.$$  

Product Grade Limit Constraint. This constraint ensures the grade limits for products are satisfied:

$$\sum_{f=1}^{F} G_{g,k,f,t} - GU_{q,d,k} \sum_{f=1}^{F} XR_{k,f,t} \leq 0 \quad \forall g,d,k,t.$$  

$$\sum_{f=1}^{F} G_{g,k,f,t} - GL_{g,d,k} \sum_{f=1}^{F} XR_{k,f,t} \geq 0 \quad \forall g,d,k,t.$$  

Flow Balance Constraints

These constraints ensure what is flowing into a node equals what is coming out of a node. This constraint has been implemented at various stages of the model and can be seen below.

Total parcel tonnage mined equals total mining from bins within that parcel:

$$X_{p,t} = \sum_{b=1}^{B} X_{p,b,t} \quad \forall p,t.$$  

Total mining from a bin equals the total flow through all paths:

$$X_{p,b,t} = \sum_{f=1}^{F} X_{p,b,f,t} \quad \forall p,b,t.$$  

Total flow through a path equals the total flow through all parcels and bins:

$$X_{f,t} = \sum_{p=1,b=1}^{P,B} X_{p,b,f,t} \quad \forall f,t.$$  

Total tonnage through each circuit equals the sum for all flow paths through that plant and circuit:

$$X_{l,c,t} = \sum_{f=1|e|f|e|f}^{F} X_{f,t} \quad \forall f,c,t.$$  

The tonnage processed through an option equals the sum of all flow paths which pass through:

$$X_{l,t} = \sum_{f=1|e|f}^{F} X_{f,t} \quad \forall l,t.$$
Total tonnage flowing into a stockpile equals the sum from all flow paths which pass through:

\[ X_{I_s,n,t} = \sum_{p,b,F} X_{p,b,f,t} \quad \forall s,n,t \big| G_{p,b} \geq GL_n \quad \text{and} \quad G_{p,b} < GU_n. \]

Total tonnage out of the stockpile equals the tonnage of each flow path out of the stockpile:

\[ X_{O_s,n,t} = \sum_{f=1}^{F} X_{f,t} \quad \forall s,n,t. \]

Total product component sale quantity may equal the total recovered component tonnage from flow paths:

\[ S_{d,k,t} = \sum_{f=1}^{F} X_{R_k,f,t} \quad \forall d,k,t \]

or the metal units recovered from flow paths:

\[ S_{d,k,t} = \sum_{f=1}^{F} G_{g,k,f,t} \quad \forall d,k,t. \]

**Non-Negativity and Integrality Constraint**

This constraint enforces non-negativity and integrality of the variables, as appropriate:

\[ X_{p,b,f,t}, X_{p,b,t}, X_{f,t}, X_{R_k,f,t}, X_{I_s,n,t}, X_{I_c,t}, X_{I_l,t}, S_{d,k,t} \geq 0 \quad \forall p,b,s,n,k,f,t,l,c,d, \]

\[ ID_{l,t} \quad \forall l,t \quad \text{and} \quad Y_{l,t}, Y_{p,t} \quad \forall p,l,t. \]

**MODEL APPLICATION AND VALIDATION**

Data from a copper-gold deposit are used to implement the methodology. This case study examines the use of several different mining capacity and plant capacity options for the deposit. It is assumed that the deposit will be mined by a single open cut operation. The model allows for dynamic configuration of the mining and plant capacities in each period.

The deposit was divided into four pushbacks generated by a single deterministic optimization. Whilst this may be considered to be removing the optimality from the model upfront, it was primarily used as a starting point for parcel separation. Likewise, the purpose of this case study is to examine the execution of mining and plant options more so than generate an optimal schedule. Each pushback was considered to be a parcel which defines a scheduling constraint in the model, i.e. parcel one (or pushback one in this case) must be mined before parcel two. Table 1 provides a summary of the deposit used in this case study. This case study uses a single resource model, however multiple stochastic models could be included in the analysis if desired (each simulation would choose one model at random).

**TABLE 1. Summary of Parcels in Resource Model**

<table>
<thead>
<tr>
<th>Parcel</th>
<th>Bins</th>
<th>Type</th>
<th>Tonnage</th>
<th>Copper, %</th>
<th>Gold, g/t</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>37</td>
<td>Ore</td>
<td>36.9</td>
<td>1.3</td>
<td>0.7</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>Waste</td>
<td>40.9</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>2</td>
<td>29</td>
<td>Ore</td>
<td>28.8</td>
<td>1.4</td>
<td>0.7</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>Waste</td>
<td>63.6</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
<td>Ore</td>
<td>7.3</td>
<td>1.4</td>
<td>0.8</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>Waste</td>
<td>21.7</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>4</td>
<td>18</td>
<td>Ore</td>
<td>17.0</td>
<td>1.2</td>
<td>0.8</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>Waste</td>
<td>66.5</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>
Several options were included in this case study to undertake a preliminary analysis of the deposit. The options used in the case study could go through further refinement after the model is processed to help guide the decision making process. For this run of the model four mining options, four processing options and two stockpiling options were examined. A summary of the options included in the model is outlined in Table 2.

The case study has used commodity price for gold and copper, capital cost, operating cost and plant utilization. No detailed analysis of the underlying nature of the stochastic variables has been carried out, as detailed research in other papers is available which was not the primary purpose of this paper [19, 7, 20, 21, 1].

The following distributions are used:

- Gold Price: Lognormal Distribution with a mean of $1000/oz and standard deviation of $550/oz;
- Copper Price: Lognormal Distribution with a mean of $5000/t and standard deviation of $4000/t;
- Capital Cost Multiple: Triangular Distribution with a lower limit of 0.5 mid point of 1.03 and upper limit of 1.35;
- Operating Cost Multiple: Triangular Distribution with a lower limit of 0.5 mid point of 1.07 and upper limit of 1.35; and
- Plant Utilization: Triangular Distribution with a lower limit of 0 %, midpoint of 85 % and upper limit of 95 %.

A sample of the gold and copper prices for the 132 simulations run is provided in Fig. 2.
One product was produced for this case study, a copper gold concentrate to be sent to a smelter. An initial run of the model determined that the model was highly sensitive to the capacity constraint placed on the product produced. With the constraint too high say 500 Mtpa all production would occur in the highest price environment for one period with subsequent periods being used to dispose of the asset. It was therefore determined that a lower capacity constraint was needed in order to generate a more constant production profile. In reality, this could represent different marketing strategies, thus, effectively capturing the supply and demand constraints.

A comprehensive model validation process was undertaken to prove the model represented reality. Several aspects were examined for model validity which included; examining the revenue, costs and option execution each period, a manual balancing of the stockpiles, examining where material flows through the network and examining the resource depletion rate. Multiple tests of the above were carried out on the data set and it was determined that the model was behaving as intended.

RESULTS ANALYSIS

A results analysis process was undertaken to help guide the decision making process. Some of the techniques used include examining frequency of execution of options, value-at-risk graphs and data mining techniques. Each of these techniques helps in a differing aspect of the decision making process.

Frequency of execution is calculated as the number of times an option is executed divided by the total number of times the option could have been executed in the timeframe. This statistic enables the decision maker to clearly see how many times different options are used over the analysis period. With this information the decision maker can determine which options are used more frequently, therefore are ‘interesting’ to the design. Based on this possible development of slightly different option designs closer to those more frequently used may help to provide a more robust mine design.

For the case study above the frequency of execution for all options is listed in Fig. 3. This shows the sum of times options were executed over the timeframe of the project for all simulations divided by the number of simulations run.

As seen in Fig. 3 three options are of particular interest. The plant option with a total capacity of 15 Mtpa is used in every design generated. Therefore, it is safe to say that of the four plant options/capacities, 15 Mtpa is the most robust. Also, the modular mining option with a capacity of 10 Mtpa is used over 30% of the time, with the expansion option for this modularity executed in nearly every simulation. This would suggest that the proposal to install 25 Mtpa would be a good consideration. Finally, the mining option with 15 Mtpa capacity is installed around 67% of the time.

Further to this, the frequency of execution for a specific option can be calculated as shown in Fig. 4. This shows the frequency of execution of the processing plant with 15 Mtpa capacity in each time period.

Also, for the modular mining option the frequency of execution was calculated across time as shown in Fig. 5. As can be seen, the option was no executed in the first time period as a constraint exist forcing the initial mine option of 10 Mtpa to be built before this modular option can be built. Also, the dynamic nature of the model is highlighted by the varying time periods in which the option is executed.

A value at risk graph (VARG) shows the risk to return relationship. Figure 6 shows the value-at-risk graph for the case study listed above. The line to the right (1) is for the simulation run with optionality enabled (that is a flexible mine design) and the line to the left (2) presents a fixed design for
the simulation run (that is no optionality). The fixed design used in the case was the 50th percentile case determined from the simulation run with optionality enabled. It would be expected that the fixed design has a lower potential project value due to its inability to change in response to market conditions. The straight lines represent the mean project value for all simulations, the mean NPV for the optionality case is $2.4 billion and for the fixed design is $2.2 billion. Therefore, by including optionality in the decision making process an increase in NPV of 11% is achieved.

Fig. 3. Frequency of execution for all options

Fig. 4. Frequency of execution for the processing plant option with 15 Mtpa capacity

Fig. 5. Frequency of execution for the modular mining option with 15 Mtpa capacity
It was determined from the results analysis that the case study had rather limited optionality, which impacted on the potential for significant NPV increase. In order to achieve further NPV increment, some of the areas that could be incorporated are:

- Multiple mining zones;
- A wider range of options for both mining and processing plants;
- Different processing routes and options, i.e. mine to crusher to conveyor to plant to conveyor to port;
- More stochasticity in the underlying variables;
- Dependant and modular option designs;
- Different potential products.

Further analysis using data mining has been investigated on other models which have been processed, but to date has not been applied to this dataset. An example of the results obtained from data mining is shown in Fig. 7. This method produces a decision tree with “go/no-go decisions” which can be used to guide decisions.
CONCLUSIONS

The paper presents a new methodology to evaluate the strategic mine design flexibility under a stochastic environment. This is achieved by developing a mixed integer programming model that determines the optimal design for simulated stochastic parameters. Generated results from the developed model can guide the decision makers in the direction of a better mine design. Increasing flexibility in mine designs would be advantageous for responding to changing business conditions across the full economic cycle. The proposed methodology has been implemented to a copper-gold deposit. The results demonstrated that the value of expected NPV increases by 11% with flexible mine design compared to inflexible. This NPV differences could be improved further by increasing the available flexibilities in the design as well as other uncertainties in the model.

Mining projects are designed on the basis of variables that are subject to extreme uncertainty. The paper illustrates how to incorporate design options (flexibility) into a strategic mine plan in a manner that proactively manages inevitable uncertainties. It is hoped this research will help in justifying more flexible mine designs and further the sustainability of the industry. Further research and model improvements continue in the following areas:

- Handling of grade variability through the use of conditional simulation methods will greatly improve the power of the model [19];
- Further investigation into appropriate results analysis techniques is required to fully understand how the primary question of flexibility is answered;
- Application of this technique to underground mining is needed to fully capture the options available to mine management. In particular, incorporating the process to optimize the open cut and underground transition point would be beneficial. This would allow strategic planning for the entire orebody.

REFERENCES